Predator-Prey Dynamics: A Model for Competition, Regulation, and Strategic Planning

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ARC

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Updated Title of Talk

- Agent-based modeling (ABM)
  - Still relevant, but not part of this talk
  - Leave it to others
- Competition
  - P-P modeling framework is at least as relevant for regulation, strategic planning, etc.

Personal Approach to ARC

- Past talks
  - Complex adaptive systems
  - Behavioral economics
  - Hyperbolic discounting
  - Predator-prey dynamics

➔ Interesting potential areas of application to actuarial science / risk management

Background

- Complex adaptive systems
  - Built bottom-up from individual agents
  - Self-organization and emergence
  - Irreducible
- Evolutionary processes
  - Parallels between economic systems and biological evolutionary theory
    - Complex systems, self-organization, emergence, adaptation (natural selection)

Background (cont.): Economics and Biology

"The precise mathematical relationship which describes the link between the frequency and size of the extinction of companies, for example, is virtually identical to that which describes the extinction of biological species in the fossil record. Only the timescales differ."

- Ormerod (2005)

Background (cont.): "Business Ecosystem"

- Economic community
- Interacting organizations and individuals
- Customers and competitors
- Leadership companies ("the keystone species")
  - Influence the co-evolutionary processes

(Moore, 1996)

Background (cont.): Economy = Ecosystem?

- Rothschild (1990): "a capitalist economy can best be comprehended as a living ecosystem. Key phenomena observed in nature – competition, specialization, co-operation, exploitation, learning, growth, and several other – are also central at business life."

Background (cont.): Personal

- Personal perspectives
  - Practicing actuary
  - Interdisciplinarity
  - Strategic and operational
- Interest in modeling economic / financial / insurance phenomenon
  - Competition
  - Supply / demand
  - Underwriting cycles
  - Regulatory effects

Lotka-Volterra Equations

(two-species model)

- Let: X: size of **prey** population

  Y: size of **predator** population

- Differential equation system:

$$\frac{dX}{dt} = X(a - bY) = aX - bXY$$

$$\frac{dY}{dt} = -Y(c - dX) = -cY + dXY$$

Lotka-Volterra Equations (cont.)

- where: $a, b, c, d > 0$

  a = intrinsic prey growth rate

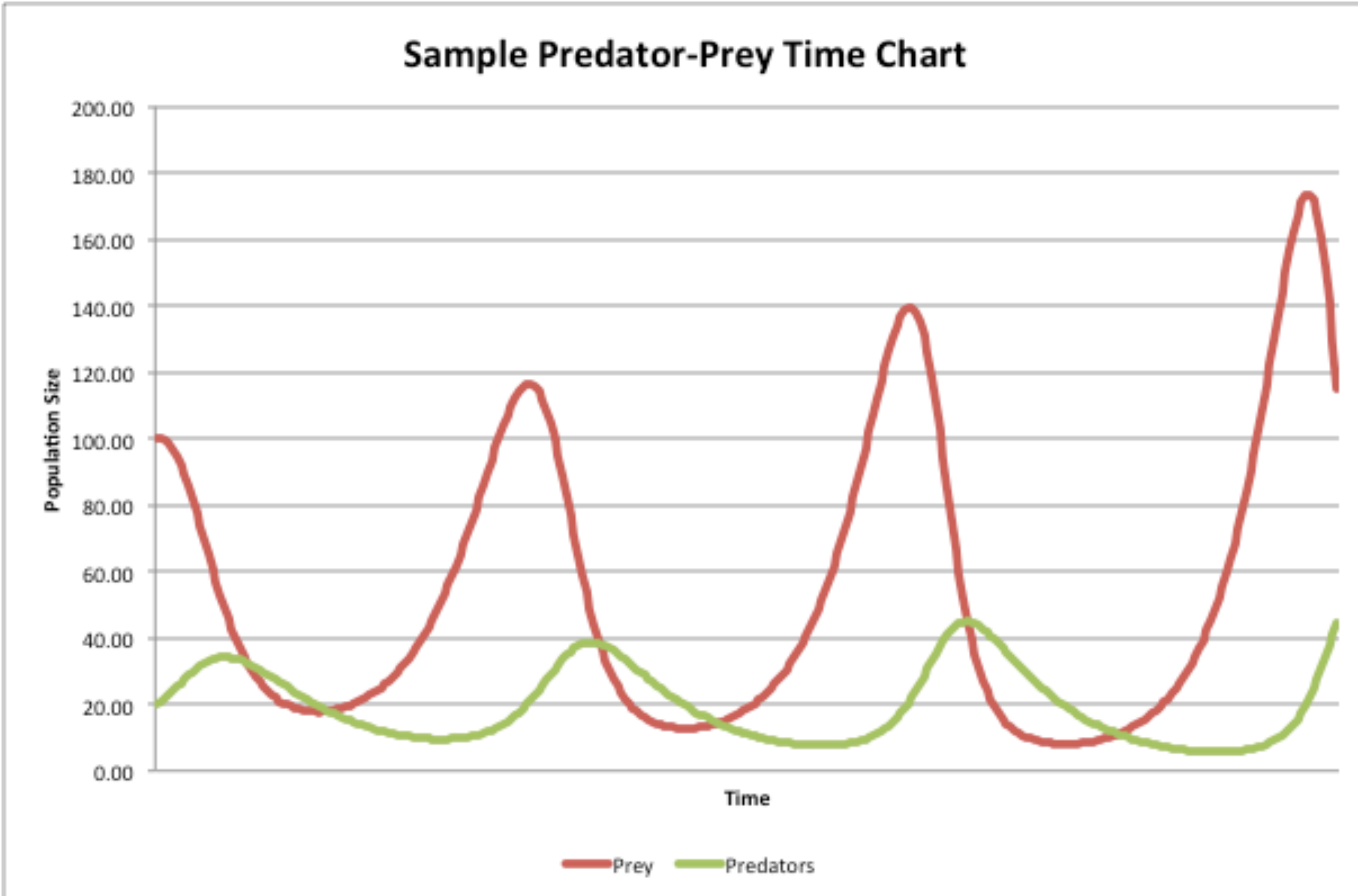
  b = predation rate coefficient

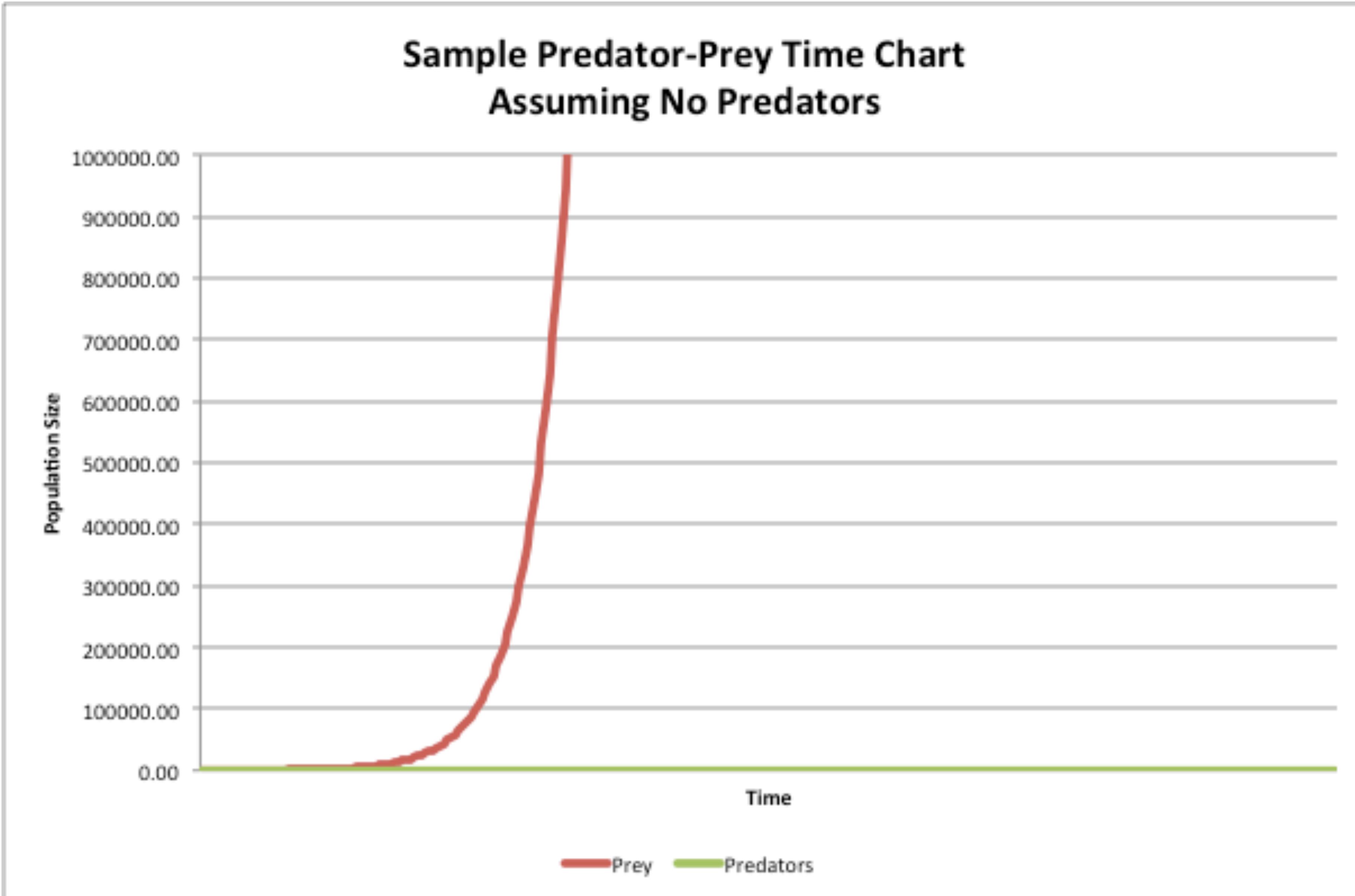
  c = predator mortality rate

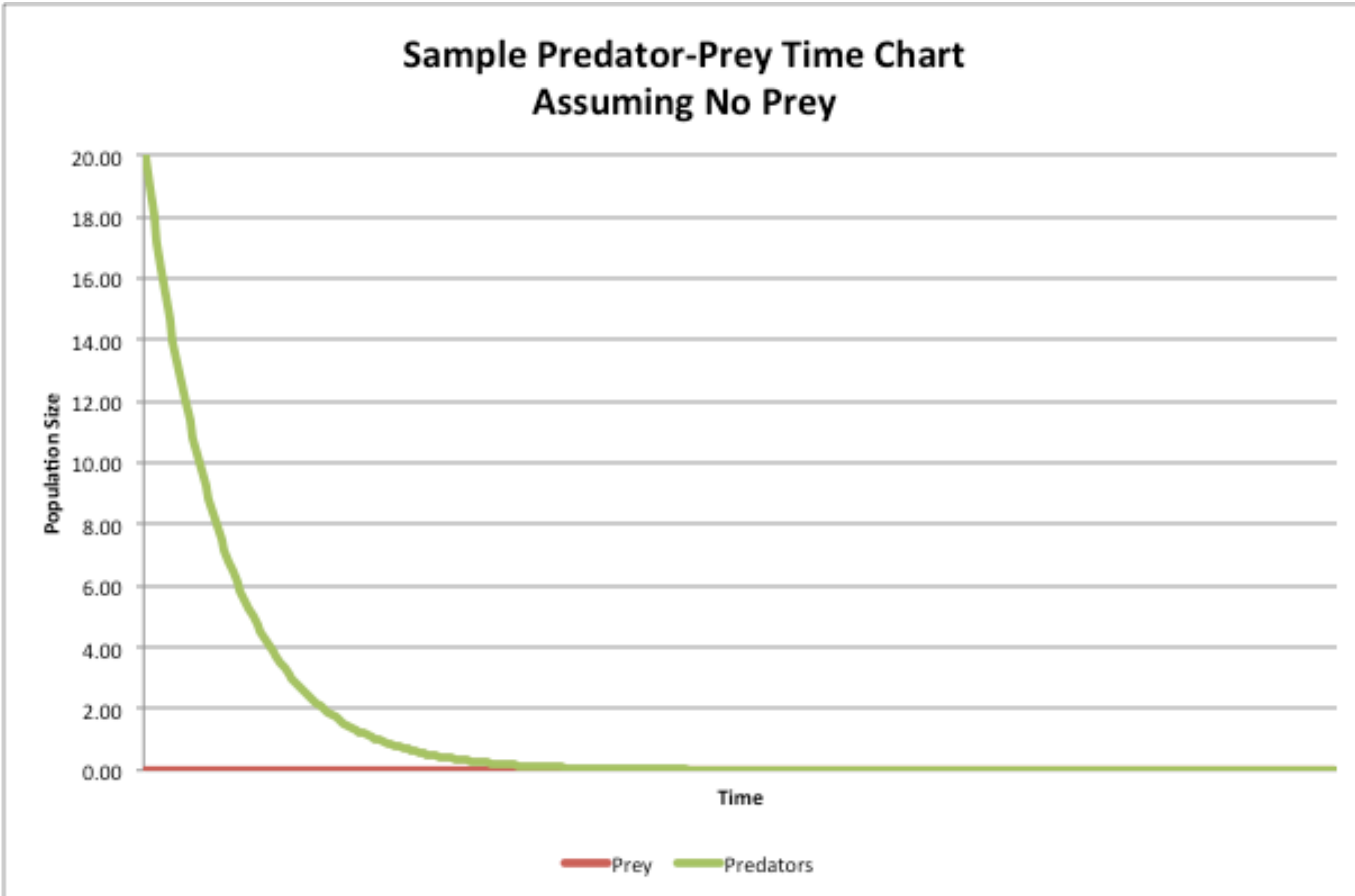
  d = predator reproduction rate per prey

Lotka-Volterra Equations (cont.)

- Assumptions underlying (two-species) L-V model:
  - Prey have access to an inexhaustible food supply
  - Prey increase exponentially in absence of predators
  - Predators feed only on prey (and thus will starve in the absence of prey)
  - No limit to amount of prey consumed by predators
  - Environment is homogeneous through time


Sample Predator-Prey Time Chart
200.00
180.00
160.00
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
Population Size
Time
Prey
Predators


Sample Predator-Prey Time Chart
Assuming No Predators
1000000.00
900000.00
800000.00
700000.00
600000.00
500000.00
400000.00
300000.00
200000.00
100000.00
0.00
Population Size
Time
Prey
Predators


Sample Predator-Prey Time Chart
Assuming No Prey
20.00
18.00
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
Population Size
Time
Prey
Predators

L-V: More Realistic

- Allow for self- (intra-) species competition
- Updated differential equation system:

$$\frac{dX}{dt} = X(a - bY - eX) = aX - bXY - eX^2$$

$$\frac{dY}{dt} = -Y(c - dX + fY) = -cY + dXY - fY^2$$

Related: The Logistic Equation

- Single process or system through time
- Differential equation:

$$\frac{dW}{dt} = gW - hW^2$$

g: reflects net birth-death rate

h: reflects competition under limited resources

Some Applications of These Models

- Prey – predator dynamic relationship:
  - Rabbits – foxes
  - Competitors in a market
  - Total product demand in a market
  - Economic fluctuations (real wages and employment)
  - Oil prices – marginal production / demand destruction / energy policy

Potential Applications in Act Sci / Ins / Risk Mgt

- Competition amongst insurers
- Pricing of insurance / other risk management vehicles
- Underwriting cycle
- Regulation

➔ Part of CAE Grant: U/G Research


Gracias


Merci


THANK YOU


Vielen
Dank


Grazie